Analysis of an infinite sequence

Assignment

Now we will expand our investigation to determine the sum of the infinite sequence tn, where

t0=1,t1=(xlna)1,t2=(xlna)22x1,t3=(xlna)33x2x1,…

Define Tn(a,x) as the sum of the first n terms, for various values of a and x.

e.g. T9(2,9) is the sum of the first nine terms when a=2 and x=5.

Let a=2. Calculate T9(2,x) for various positive values of x. Using technology, plot the relation between T9(2,x) and x. Describe what you notice from your plot.

Let a=3. Calculate T9(3,x) for various positive values of x. Using technology, plot the relation between T9(3,x) and x. Describe what you notice from your plot.

Continue with this analysis to find the general statement for Tn(a,x) as n approaches ∞.

Test the validity of the general statement with other values of a and x.

Discuss the scope and/or limitations of the general statement.

Explain how you arrived at the general statement.

Work

1. First we will use the mathematical program Derive™ 6 to calculate the sum of the first 9 terms of the sequence when a=2, for various values of x.

The expression has the following form:

SUM(Expression, summation parameter, lower boundary, upper boundary)

Which in our case makes:

SUM(((x*ln(2))^n)/n!,n,0,9)

Derive™ 6 then shows the following:

n=09(x·LN(2))nn!

Option Simplify gives us the expanded form of the sum:

x9LN(a)9362880+x8LN(a)840320+x7LN(a)75040+x6LN(a)6720+x5LN(a)5120+x4LN(a)424+
x3LN(a)36+x2LN(a)22+x·LN(a)+1

By using the button ≈, the following results were reached for arbitrarily chosen values of x:

n	a	x	Tn(a,x)
9	2	0.1	1.071773462
9	2	0.2	1.148698354
9	2	0.5	1.414213562
9	2	1	2
9	2	2	3.999991747
9	2	3	7.999488457
9	2	8	241.6244730
9	2	10	857.4734736

TABLE 1. Calculated sums of 9 terms of the sequence for various values of x when a=2

The calculated value for x=1 stands out, being an integer.

We now write a MATLAB© program to plot the sum as a function of x.

The main program code:

```
a=2;

n=9; % the number of terms contained in the sum

L=log(a); % To avoid needless calculation in each iteration,

% the natural logarithm is calculated at the beginning.

Tn=0; % Setting the initial sum of the sequence to zero.
```